Statistics
Spring 2022
Lecture 7

Multiplication Rule

1) Independent events

$P(A \text{ and } B) = P(A) \cdot P(B)$

Ex: $P(A) = .3$, $P(B) = .4$, A and B are independent events

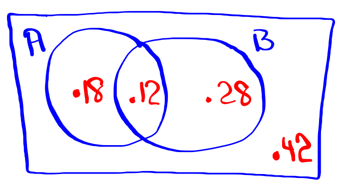
1) $P(\overline{A}) = 1 - P(A)$
$= \boxed{.7}$

2) $P(\overline{B}) = 1 - P(B)$
$= \boxed{.6}$

3) $P(A \text{ and } B) = P(A) \cdot P(B)$
$= (.3)(.4) = \boxed{.12}$

4) Make Venn Diagram

$.3 - .12 = .18$
$.4 - .12 = .28$



Total = 1

5) $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .3 + .4 - .12 = \boxed{.58}$

A box has 3 Dimes and 7 Nickels.
Randomly Select 2 Coins with replacement

Sample Space:
DD
DN
ND
NN

DD → 20¢
DN → 15¢
ND → 15¢
NN → 10¢

$P(20¢) = P(DD) = \frac{3}{10} \cdot \frac{3}{10} = \frac{9}{100} = \boxed{.09}$

$P(15¢) = P(DN \text{ or } ND) = \frac{3}{10} \cdot \frac{7}{10} + \frac{7}{10} \cdot \frac{3}{10} = \frac{42}{100} = \boxed{.42}$

$P(10¢) = P(NN) = \frac{7}{10} \cdot \frac{7}{10} = \frac{49}{100} = \boxed{.49}$

Total(¢)	P(Total(¢))
20	.09
15	.42
10	.49

Clear all lists
Total → L1
P(Total) → L2
Use 1-Var Stats with List: L1, FreqList: L2 to find

$\bar{x} = 13$
$S_x =$ Blank
$n = 1$

Suppose P(Tails) = .2 P(Heads) = .8
Toss this Coin 3 times

Sample Space:
TTT
TTH
THT
THH
HTT
HTH
HHT
HHH

P(exactly 3 Tails) =
$P(TTT) = (.2)(.2)(.2) = \boxed{.008}$

P(exactly 2 Tails) =
$3 \cdot (.2)(.2)(.8) = \boxed{.096}$

P(exactly 1 Tail) =
$3 \cdot (.2)(.8)(.8) = \boxed{.384}$

P(No tails) = P(All heads)
$= (.8)(.8)(.8) = \boxed{.512}$

# tails	P(# Tails)
3	.008
2	.096
1	.384
0	.512

Clear all lists
Tails → L1
P(# Tails) → L2
Use 1-Var Stats with List: L1, FreqList: L2 to find

$\bar{x} = .6$
$S =$ Blank
$n = 1$ ← Total Prob. = 1

2) Dependent events

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

There are 2 Females and 3 Males.

Select 2 people (without replacement)

Sample Space:

MM
MF
FM
FF

$P(\text{2 males}) = P(MM)$ → 0 Females

$= \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \boxed{.3}$

$P(1M, 1F) = P(MF \text{ or } FM)$

1 Female

$= \frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{3}{4} = \frac{12}{20} = \boxed{.6}$

$P(\text{2 Females}) = P(FF) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \boxed{.1}$

# Females	P(# Females)
0	.3
1	.6
2	.1

Clear all lists

F → L1

P(# F) → L2

1 – Var Stats

List: L1, FreqList: L2

Find

$\bar{x} = .8$

S = Blank

n = 1

Conditional Probability

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

If we solve for $P(B|A)$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

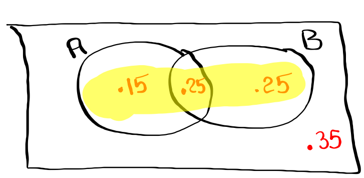
Ex: $P(A) = .4$, $P(B) = .5$, $P(A \text{ and } B) = .25$

1) Make Venn Diagram $.4 - .25 = .15$

$P(\bar{A}) = .6$

$P(\bar{B}) = .5$

$P(A \text{ or } B) = .65$



Total = 1

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.25}{.4} = \boxed{.625}$

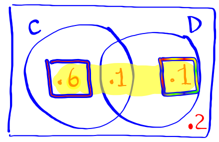
Given

$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.25}{.5} = \boxed{.5}$

P(Coffee) = .7

P(Donuts) = .2

P(Coffee and Donuts) = .1



P(Coffee or Donuts) = $\boxed{.8}$

P($\overbrace{\text{Coffee only}}$ or $\overbrace{\text{Donuts only}}$) = .6 + .1 = $\boxed{.7}$

P(Donuts | Coffee) = $\dfrac{\text{P(Coffee and Donuts)}}{\text{P(Coffee)}}$ = $\dfrac{.1}{.7}$ = $\boxed{.143}$

Given

P(Coffee | Donuts) = $\dfrac{\text{P(Coffee and Donuts)}}{\text{P(Donuts)}}$ = $\dfrac{.1}{.2}$ = $\boxed{.5}$

No class Thursday
week after next
Spring Break

Our next meeting is 3 weeks from tonight.

I will have office hrs.
You will have assignments.

Prob. with at least 1:

P(at least 1) = 1 − P(None)

Toss a fair coin 3 times.

P(H) = .5 , P(T) = .5

HHH

Some T
Some H

TTT

P(3 Tails) = (.5)(.5)(.5) = $\boxed{.125}$

P(at least 1 Tail) = 1 − P(No tails)

= 1 − P(HHH) = 1 − (.5)(.5)(.5) = $\boxed{.875}$

3 Females , 5 Males

Select 3 different people
(No replacement)

FFF

Some F
Some M

MMM

$P(3\text{ Females}) = \frac{3}{8}\cdot\frac{2}{7}\cdot\frac{1}{6} = \boxed{\frac{1}{56}}$

$P(3\text{ Males}) = \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} = \boxed{\frac{5}{28}}$

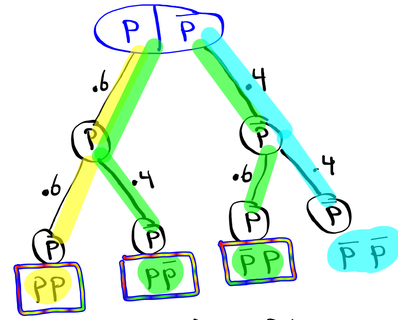
$P(\text{at least 1 Female}) = 1 - P(\text{No Females})$
$= 1 - \frac{5}{28} = \boxed{\frac{23}{28}}$

$P(\text{at least 1 male}) = 1 - P(\text{No males})$
$= 1 - P(\text{All Females})$
$= 1 - \frac{1}{56} = \boxed{\frac{55}{56}}$

Multiplication Rules with Tree Diagram.

P(passing a class) = .6

2 students randomly Selected.



P(both Pass) = (.6)(.6) = .36

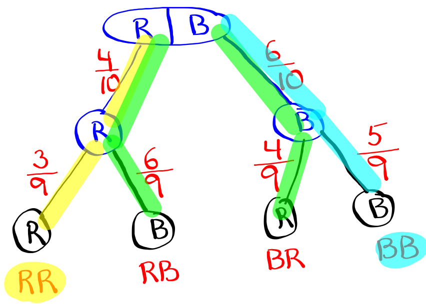
P(exactly 1 pass) = 2(.6)(.4) = .48

P(None pass) = (.4)(.4) = .16

P(at least 1 Pass) = 1 − P(None Pass)
$= 1 - .16 = \boxed{.84}$

4 Red, 6 Blue Balls

Select 2 balls No replacement



$P(\text{2 Blue Balls}) = \frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$

$P(\text{2 Red Balls}) = \frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$

$P(\text{at least 1 Blue Ball}) = 1 - P(\text{No Blue})$
$= 1 - P(RR) = 1 - \frac{2}{15} = \boxed{\frac{13}{15}}$

$P(\text{at least 1 Red Ball}) = 1 - P(\text{No Red})$
$= 1 - P(BB)$
$= 1 - \frac{1}{3} = \boxed{\frac{2}{3}}$

Counting Methods:

How many choices is you were to Select One number from 0 to 9? 10

Pick a letter, then choose a number from 0 to 9.

10

A0 A1 A2 - - - - A9
B0 B1 B2 . - - B9
⋮
Z0 Z1 Z2 . - - . Z9

26

260 choices

what is letters are case Sensetive?

10 choices

52 choices

520 choices

5 people, Select 2 of them

Adam, Bill, Carol, David, Elizabeth.

~~AB~~ ~~AC~~ ~~AD~~ ~~AE~~

BA ~~BC~~ ~~BD~~ ~~BE~~

CA CB ~~CD~~ ~~CE~~

DA DB DC ~~DE~~

EA EB EC ED

First · Second

$5 \cdot 4 = \boxed{20}$

what if order does not matter?

10 choices

Combination Formula $_5C_2 = \boxed{10}$

5 $\boxed{\text{MATH}}$ $\boxed{\rightarrow}$ $\boxed{\text{PRB}}$ $\boxed{\downarrow}$ $\boxed{_nC_r}$ 2 $\boxed{\text{Enter}}$

A basketball team has 12 players,

5 have to start the game.

How many selections?

$_{12}C_5 = 792$

12 MATH $\rightarrow$ PRB $\downarrow$ $_nC_r$ 5 $\boxed{\text{Enter}}$

CA Lotto

50 numbers, Select 5, order does not matter

How many ways? $_{50}C_5 = \boxed{2,118,760}$

4 Females, 6 Males, Select 3 people
order does not matter.

1) How many total Selections? ${}_{10}C_3 = \boxed{120}$

2) How many ways can we Select 3 Females?
${}_4C_3 = \boxed{4}$

3) P(Select 3 Females) $= \frac{{}_4C_3}{{}_{10}C_3} = \frac{4}{120} = \boxed{\frac{1}{30}}$

4) P(Select 3 Males) $= \frac{{}_6C_3}{{}_{10}C_3} = \frac{20}{120} = \boxed{\frac{1}{6}}$

5) P(at least 1 Male) = 1 − P(No male)
= 1 − P(All Females) $= \boxed{\frac{29}{30}}$

6) P(1F 2M) $= \frac{{}_4C_1 \cdot {}_6C_2}{{}_{10}C_3} = \frac{60}{120} = \boxed{\frac{1}{2}}$

7) P(2F 1M) $= \frac{{}_4C_2 \cdot {}_6C_1}{{}_{10}C_3} = \frac{36}{120} = \boxed{\frac{3}{10}}$

A deck of Cards has 52 Cards,
12 Face Cards, and 4 Aces.
Select 4 Cards, No replacement, order does not matter.

$$P(4 \text{ Face Cards}) = \frac{{}_{12}C_4}{{}_{52}C_4} = \boxed{.002}$$

$$P(4 \text{ Aces}) = \frac{{}_4C_4}{{}_{52}C_4} = \boxed{3.7\times10^{-6}}$$

$$P(2 \text{ Face } \& \text{ } 2 \text{ Aces}) = \frac{{}_{12}C_2 \cdot {}_4C_2}{{}_{52}C_4} = \boxed{.001}$$

John hired 3 Females and 12 Males.
He needs 8 Morning shift, 7 evening shift.

1) How many ways can he make the Schedule?

Morning · Evening

$${}_{15}C_8 \cdot {}_7C_7 = \boxed{6435}$$

2) Consider Morning shift only

$$P(3F \,\&\, 5M) = \frac{{}_3C_3 \cdot {}_{12}C_5}{{}_{15}C_8} = \boxed{\frac{8}{65}}$$

3) $$P(2F \text{ and } 6M) = \frac{{}_3C_2 \cdot {}_{12}C_6}{{}_{15}C_8} = \boxed{\frac{28}{65}}$$